# TWO-DIMENSIONAL UNSTEADY TEMPERATURE FIELD ON THE SURFACE OF A SEMIINFINITE BODY SUBJECT TO HEATING BY AN ANNULAR PULSE

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The development of an unsteady temperature field on the surface of a semiinfinite body is studied in the case where the body is heated by rectangular heat pulse through an annular region.

The heating of infinite and semiinfinite bodies by short pulses of heat through regions of various shapes is widely used in many of the pulse methods of determining the thermal and physical characteristics of materials [1-3].

In the present paper we present an analytical method of nondestructive analysis of the thermal and physical characteristics of the materials. The heating of the surface of the body is supplied by a thin annular heat source with a heat flux density given by

$$q(\tau) = q_0 U(\tau_0 - \tau).$$

The problem is formulated as follows. The semiinfinite body has an initial temperature $T_0$ = const. For times $\tau > 0$ part of the surface (an annular heat source) is heated by a pulse heat flux of density $q_0$ and duration $\tau_0$. The rest of the surface, i.e., the region $0 \leq r \leq R_1$ and $R_2 < r < \infty$, is thermally insulated. The origin of coordinates $r = z = 0$ is chosen to be the center of the annular region on the surface ($z = 0$) of the body.

The above statement of the problem leads to a system of three heat-conduction equations

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial\Theta_i(r, z, \tau)}{\partial r}\right] + \frac{\partial^2\Theta_i(r, z, \tau)}{\partial z^2} = \frac{1}{a}\frac{\partial\Theta_i(r, z, \tau)}{\partial\tau} \quad (i=1, 2, 3), \tag{1}$$

where $\Theta_1(r, z, \tau)$ is the excess temperature in the region $0 \leq r < R_1$, $z \geq 0$, $\tau > 0$; $\Theta_2(r, z, \tau)$ – $R_1 \leq r \leq R_2$, $z \geq 0$, $\tau > 0$; $\Theta_3(r, z, \tau)$ – $r > R_2$, $z \geq 0$, $\tau > 0$.

The boundary conditions for (1) are written in the form

$$-\lambda\frac{\partial\Theta_2(r, 0, \tau)}{\partial z}\bigg|_{\substack{z=0,\\ R_1<r<R_2}} = \begin{cases} q_0 = \text{const} & \text{for } \tau < \tau_0, \\ 0 & \text{for } \tau > \tau_0; \end{cases} \tag{2}$$

$$\frac{\partial\Theta_1(r, 0, \tau)}{\partial z} = \frac{\partial\Theta_3(r, 0, \tau)}{\partial z} = \frac{\partial\Theta_1(0, z, \tau)}{\partial r} = 0; \tag{3}$$

$$\frac{\partial\Theta_i(r, \infty, \tau)}{\partial z} = 0; \tag{4}$$

$$\frac{\partial\Theta_3(\infty, z, \tau)}{\partial r} = 0; \tag{5}$$

$$\Theta_1(R_1, z, \tau) = \Theta_2(R_1, z, \tau); \tag{6}$$

$$\frac{\partial\Theta_1(R_1, z, \tau)}{\partial r} = \frac{\partial\Theta_2(R_1, z, \tau)}{\partial r}; \tag{7}$$

$$\Theta_i(r, z, 0) = 0; \tag{8}$$

$$\Theta_2(R_2, z, \tau) = \Theta_3(R_2, z, \tau); \tag{9}$$

$$\frac{\partial\Theta_2(R_2, z, \tau)}{\partial r} = \frac{\partial\Theta_3(R_2, z, \tau)}{\partial r}. \tag{10}$$

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The solution of system (1) with the boundary conditions (2)-(10) for any point on the surface z = 0 of the semiinfinite body is written for the above three regions of r:

for $0 \leq r < R_1$, $z = 0$, $\tau > 0$ (Fo > 0)

$$\frac{\Theta_1^*(r,\,0,\,\mathrm{Fo})}{\mathrm{Ki}} = \sqrt{\frac{2}{\pi}}\sum_{n=0}^{\infty}\sum_{m=0}^{n} A_{n,m}\left(\frac{r}{R_2}\right)^{2n}\mathrm{Fo}^{-n+\frac{m}{2}+\frac{3}{4}}\times$$
$$\times\left\{K_R^{-m-\frac{1}{2}}\exp\left[-\frac{K_R^2}{8\mathrm{Fo}}\right]W_{n-\frac{m}{2}-\frac{3}{4},\,\frac{m}{2}-\frac{1}{4}}\left(\frac{K_R^2}{4\mathrm{Fo}}\right)-\right.$$
$$-\exp\left[-\frac{1}{8\mathrm{Fo}}\right]W_{n-\frac{m}{2}-\frac{3}{4},\,\frac{m}{2}-\frac{1}{4}}\left(\frac{1}{4\mathrm{Fo}}\right)-U(\mathrm{Fo}-\mathrm{Fo}_0)\times \tag{11}$$
$$\times(1-\varphi_0)^{-n+\frac{m}{2}+\frac{3}{4}}\left[K_R^{-m-\frac{1}{2}}\exp\left[-\frac{K_R^2}{8\mathrm{Fo}(1-\varphi_0)}\right]\times\right.$$
$$\left.\left.\times W_{n-\frac{m}{2}-\frac{3}{4},\,\frac{m}{2}-\frac{1}{4}}\left(\frac{K_R^2}{4\mathrm{Fo}(1-\varphi_0)}\right)-\exp\left[-\frac{1}{8\mathrm{Fo}(1-\varphi_0)}\right]W_{n-\frac{m}{2}-\frac{3}{4},\,\frac{m}{2}-\frac{1}{4}}\left(\frac{1}{4\mathrm{Fo}(1-\varphi_0)}\right)\right]\right\};$$

for $R_1 < r < R_2$, $z = 0$, $\tau > 0$ (Fo > 0)

$$\frac{\Theta_2^*(r,\,0,\,\mathrm{Fo})}{\mathrm{Ki}} = \frac{2}{\sqrt{\pi}}\sqrt{\mathrm{Fo}}\left[1-U(\mathrm{Fo}-\mathrm{Fo}_0)\sqrt{1-\varphi_0}\right]-\sqrt{\frac{2}{\pi}}\times$$
$$\times\sum_{n=0}^{\infty}\sum_{m=0}^{n}A_{n,m}\left(\frac{r}{R_2}\right)^{2n}\mathrm{Fo}^{-n+\frac{m}{2}+\frac{3}{4}}\left[\exp\left(-\frac{1}{8\,\mathrm{Fo}}\right]\times\right.$$
$$\times W_{n-\frac{m}{2}-\frac{3}{4},\,\frac{m}{2}-\frac{1}{4}}\left(\frac{1}{4\,\mathrm{Fo}}\right)-U(\mathrm{Fo}-\mathrm{Fo}_0)(1-\varphi_0)^{-n+\frac{m}{2}+\frac{3}{4}}\times$$
$$\left.\times\exp\left[-\frac{1}{8\mathrm{Fo}(1-\varphi_0)}\right]W_{n-\frac{m}{2}-\frac{3}{4},\,\frac{m}{2}-\frac{1}{4}}\left(\frac{1}{4\mathrm{Fo}(1-\varphi_0)}\right)\right]- \tag{12}$$
$$-\sqrt{\frac{2}{\pi}}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{A_{n,m}}{2(n+1)}\left(\frac{r}{R_2}\right)^{-m-\frac{3}{2}}K_R^{2n+2}\,\mathrm{Fo}^{-n+\frac{m}{2}+\frac{1}{4}}\times$$
$$\times\left[\exp\left[-\frac{(r/R_2)^2}{8\mathrm{Fo}}\right]W_{n-\frac{m}{2}-\frac{1}{4},\,\frac{m}{2}+\frac{1}{4}}\left(\frac{(r/R_2)^2}{4\mathrm{Fo}}\right)-U(\mathrm{Fo}-\mathrm{Fo}_0)\times\right.$$
$$\left.\times(1-\varphi_0)^{-n+\frac{m}{2}+\frac{1}{4}}\exp\left[-\frac{(r/R_2)^2}{8\mathrm{Fo}(1-\varphi_0)}\right]W_{n-\frac{m}{2}-\frac{1}{4},\,\frac{m}{2}+\frac{1}{4}}\left(\frac{(r/R_2)^2}{4\mathrm{Fo}(1-\varphi_0)}\right)\right];$$

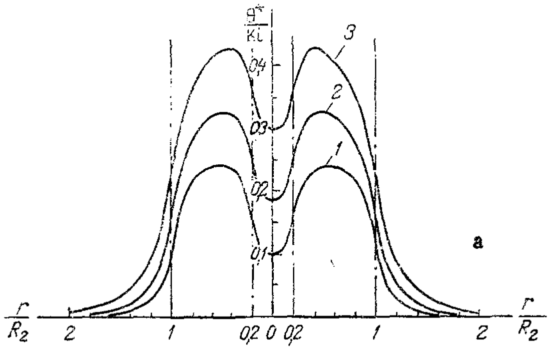
for $R_2 < r < \infty$, $z = 0$, $\tau > 0$ (Fo > 0)

$$\frac{\Theta_3^*(r,\,0,\,\mathrm{Fo})}{\mathrm{Ki}} = \sqrt{\frac{2}{\pi}}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{A_{n,m}}{2(n+1)}\left[1-K_R^{2n+2}\right]\left(\frac{r}{R_2}\right)^{-m-\frac{3}{2}}\times$$
$$\times\mathrm{Fo}^{-n+\frac{m}{2}+\frac{1}{4}}\left[\exp\left[-\frac{(r/R_2)^2}{8\mathrm{Fo}}\right]W_{n-\frac{m}{2}-\frac{1}{4},\,\frac{m}{2}+\frac{1}{4}}\left(\frac{(r/R_2)^2}{4\mathrm{Fo}}\right)-\right. \tag{13}$$
$$\left.-U(\mathrm{Fo}-\mathrm{Fo}_0)(1-\varphi_0)^{-n+\frac{m}{2}+\frac{1}{4}}\exp\left[-\frac{(r/R_2)^2}{8\mathrm{Fo}(1-\varphi_0)}\right]W_{n-\frac{m}{2}-\frac{1}{4},\,\frac{m}{2}+\frac{1}{4}}\left(\frac{(r/R_2)^2}{4\mathrm{Fo}(1-\varphi_0)}\right)\right],$$

where

$$A_{n,m} = \frac{C_n^m\left(\frac{1}{2}\right)_m 2^m}{4^n\,(n!)^2}. \tag{14}$$

When $\mathrm{Fo} \leq \mathrm{Fo}_0$ ($\tau_0 \geq \tau$) it is not difficult to obtain from (11)-(13) expressions for the relative excess temperatures $\Theta_i^*$ (r, 0, Fo)/Ki for the case of a continuous (constant) heat



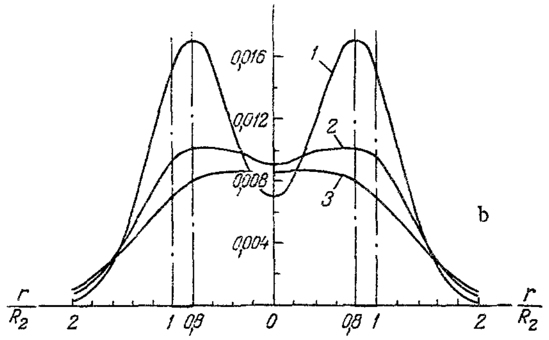




Fig. 1. Dependence of the dimensionless temperature on the relative coordinate $r/R_2$ for fixed values of Fo on the surface of a semiinfinite body: a, Fo < $Fo_0$, $K_R$ = 0.2; 1) Fo = 0.05; 2) 0.1; 3) Fo = 0.2; b, Fo > $Fo_0$, $K_R$ = 0.8, $Fo_0$ = 0.04; 1) Fo = 0.1; 2) 0.15; 3) Fo = 0.2.

flux $q_0$ = const in an annular region $R_1 \leqq r \leqq R_2$ on the surface of the semiinfinite body, since in this case $U(Fo - Fo_0) = 0$.

In the general case (Fo > $Fo_0$) the temperature fields (11) through (13) on the surface of the semiinfinite body, subject to a heat pulse on the surface, are characterized by two stages:

a) In the first stage ($0 < \tau \leqq \tau_0$) there is an unsteady heating of the semiinfinite body and the temperature of the surface rises to the values given by (11)-(13).

b) In the final stage (upon the termination of the source $q_0$ = const for $\tau > \tau_0$ (Fo > $Fo_0$)) there is a redistribution of the heat absorbed by the body in the first stage and there is a slow equalization of the temperatures (11)-(13) over the entire volume of the body.

Figure 1 shows graphs of the relative excess temperatures (11)-(13) for these two stages, as functions of the relative coordinate $r/R_2$ for fixed values of the parameters Fo, $\varphi_0$, and $K_R$.

We consider the solution (11)-(13) for limiting values of $R_1$ and $R_2$ of a thin annular heater with the heat flux (2). When $R_1 \to 0$ we obtain from (12) and (13) the unsteady temperature distribution on the surface of the semiinfinite body for the case of a thin circular heat source of radius $R_2$ and duration $\tau_0$ [9, 10]:

$$\lim_{\substack{K_R\to 0\\(R_1\to 0)}} \frac{\Theta_2^*(r,\ 0,\ \mathrm{Fo})}{\mathrm{Ki}} = \frac{\overline{\Theta}_2^*(r,\ 0,\ \mathrm{Fo})}{\mathrm{Ki}} = \frac{2}{\sqrt{\pi}}\sqrt{\mathrm{Fo}}\,[1-U(\mathrm{Fo}-\mathrm{Fo}_0)\times \times\sqrt{1-\varphi_0}] - \sqrt{\frac{2}{\pi}}\sum_{n=0}^{\infty}\sum_{m=0}^{n} A_{n,m}\left(\frac{r}{R_2}\right)^{2n}\mathrm{Fo}^{-n+\frac{m}{2}+\frac{3}{4}}\times \times\left[\exp\left(-\frac{1}{8\mathrm{Fo}}\right)W_{n-\frac{m}{2}-\frac{3}{4},\ \frac{m}{2}-\frac{1}{4}}\left(\frac{1}{4\mathrm{Fo}}\right) - U(\mathrm{Fo}-\mathrm{Fo}_0)\times \times(1-\varphi_0)^{-n+\frac{m}{2}+\frac{3}{4}}\exp\left[-\frac{1}{8\mathrm{Fo}(1-\varphi_0)}\right]W_{n-\frac{m}{2}-\frac{3}{4},\ \frac{m}{2}-\frac{1}{4}}\left(\frac{1}{4\mathrm{Fo}(1-\varphi_0)}\right)\right]; \tag{15}$$

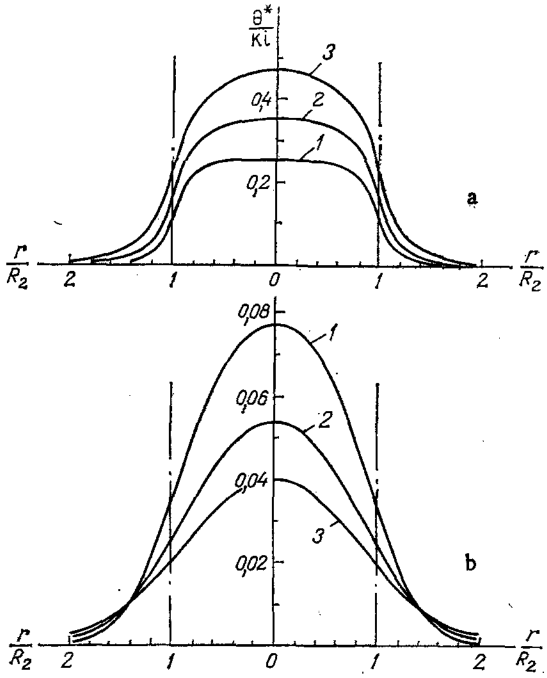



Fig. 2. Dependence of the dimensionless temperature on the relative coordinate $r/R_2$ for $R_1 = 0$ and fixed Fo on the surface of a semiinfinite body: a, $\mathrm{Fo} < \mathrm{Fo}_0$; 1) $\mathrm{Fo} = 0.05$; 2) 0.1; 3) $\mathrm{Fo} = 0.2$; b, $\mathrm{Fo} > \mathrm{Fo}_0$; $\mathrm{Fe}_0 = 0.04$, 1) $\mathrm{Fo} = 0.1$; 2) 0.15; 3) $\mathrm{Fo} = 0.2$.

$$\lim_{\substack{K_R\to 0\\(R_1\to 0)}} \frac{\Theta_3^*(r,\ 0,\ \mathrm{Fo})}{\mathrm{Ki}} = \frac{\overline{\Theta}_3^*(r,\ 0,\ \mathrm{Fo})}{\mathrm{Ki}} = \sqrt{\frac{2}{\pi}} \sum_{n=0}^{\infty}\sum_{m=0}^{n} \frac{A_{n,m}}{2(n+1)} \left(\frac{r}{R_2}\right)^{-m-\frac{3}{2}} \mathrm{Fo}^{-n+\frac{m}{2}+\frac{1}{4}} \left[\exp\left[-\frac{(r/R_2)^2}{8\mathrm{Fo}}\right]\times\right.$$
$$\times W_{n-\frac{m}{2}-\frac{1}{4},\ \frac{m}{2}+\frac{1}{4}}\left(\frac{(r/R_2)^2}{4\mathrm{Fo}}\right) - U(\mathrm{Fo}-\mathrm{Fo}_0)(1-\varphi_0)^{-n+\frac{m}{2}+\frac{1}{4}}\times \quad (16)$$
$$\left.\times \exp\left[-\frac{(r/R_2)^2}{8\mathrm{Fo}(1-\varphi_0)}\right] W_{n-\frac{m}{2}-\frac{1}{4},\ \frac{m}{2}+\frac{1}{4}}\left(\frac{(r/R_2)^2}{4\mathrm{Fo}(1-\varphi_0)}\right)\right].$$

The unsteady temperature distributions (15) and (16) for a circular heat pulse acting on the surface are shown in Fig. 2 for the two stages ($\mathrm{Fo} \leq \mathrm{Fo}_0$ and $\mathrm{Fo} > \mathrm{Fo}_0$) as functions of the relative radius $r/R_2$ for fixed values of Fo and $\varphi_0$.

When $R_2 \to \infty$ ($K_R \to 0$) we obtain from (11) and (12) the unsteady temperature fields on the surface for a rectangular heat pulse $q_0 = \mathrm{const}$ of duration $\tau_0$ acting over the entire surface of the body except for the regions $0 \leq r < R_1$ ($z = 0$):

$$\lim_{\substack{K_R\to 0\\(R_2\to \infty)}} \frac{\Theta_1^*(r,\ 0,\ \mathrm{Fo})}{\mathrm{Ki}} = \frac{\overline{\Theta}_1^*(r,\ 0,\ \mathrm{Fo}_1)}{\mathrm{Ki}_1} = \sqrt{\frac{2}{\pi}} \sum_{n=0}^{\infty}\sum_{m=0}^{n} A_{n,m}\left(\frac{r}{R_1}\right)^{2n}\times$$
$$\times \mathrm{Fo}_1^{-n+\frac{m}{2}+\frac{3}{4}}\left[\exp\left(-\frac{1}{8\mathrm{Fo}_1}\right) W_{n-\frac{m}{2}-\frac{3}{4},\ \frac{m}{2}-\frac{1}{4}}\left(\frac{1}{4\mathrm{Fo}_1}\right)-\right.$$
$$- U(\mathrm{Fo}_1-\mathrm{Fo}_0^*)(1-\varphi_0)^{-n+\frac{m}{2}+\frac{3}{4}} \exp\left[-\frac{1}{8\mathrm{Fo}_1(1-\varphi_0)}\right]\times \quad (17)$$
$$\left.\times W_{n-\frac{m}{2}-\frac{3}{4},\ \frac{m}{2}-\frac{1}{4}}\left(\frac{1}{4\mathrm{Fo}_1(1-\varphi_0)}\right)\right];$$

$$\lim_{\substack{K_R\to 0\\(R_2\to \infty)}} \frac{\Theta_2^*(r,\ 0,\ \mathrm{Fo})}{\mathrm{Ki}} = \frac{\overline{\Theta}_2^*(r,\ 0,\ \mathrm{Fo}_1)}{\mathrm{Ki}_1} = \frac{2\sqrt{\mathrm{Fo}_1}}{\sqrt{\pi}}\,[1-U(\mathrm{Fo}_1-\mathrm{Fo}_0^*)\times \quad (18)$$
$$\sqrt{1-\varphi_0}] - \sqrt{\frac{2}{\pi}} \sum_{n=0}^{\infty}\sum_{m=0}^{n} \frac{A_{n,m}}{2(n+1)}\left(\frac{r}{R_1}\right)^{-m-\frac{3}{2}} \mathrm{Fo}_1^{-n+\frac{m}{2}+\frac{1}{4}}\times$$

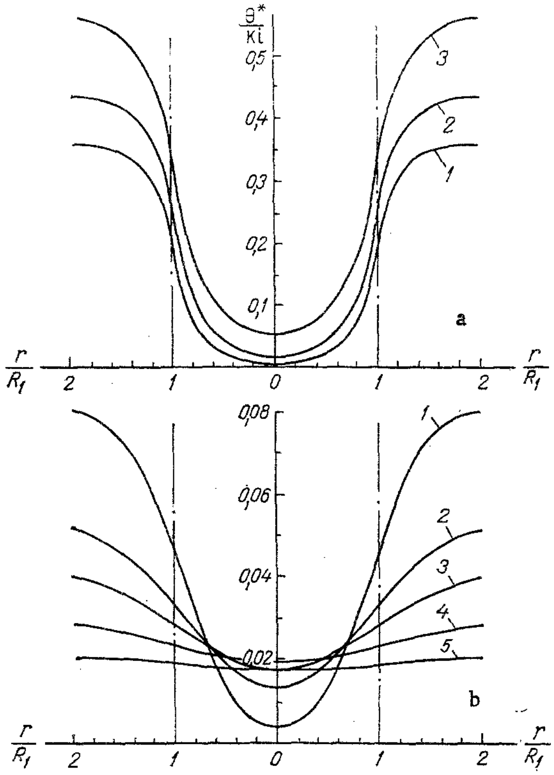



Fig. 3. Dependence of the dimensionless temperature on the relative coordinate $r/R_1$ for $R_2 = \infty$ and fixed values of Fo on the surface of a semiinfinite body: a, $Fo_1 < Fo_0^*$; 1) $Fo_1 = 0.1$, 2) 0.15; 3) $Fo_1 = 0.25$; b, $Fo_1 > Fo_0^*$, $Fo_0^* = 0.04$; 1) $Fo_1 = 0.1$; 2) 0.2; 3) 0.3; 4) 0.54; 5) $Fo_1 = 1.0$.

$$\times \left[\exp\left[-\frac{(r/R_1)^2}{8Fo_1}\right] W_{n-\frac{m}{2}-\frac{1}{4},\frac{m}{2}+\frac{1}{4}}\left(\frac{(r/R_1)^2}{4Fo_1}\right) - U(Fo_1 - Fo_0^*)\right.$$

$$\left.(1-\varphi_0)^{-n+\frac{m}{2}+\frac{1}{4}} \exp\left[-\frac{(r/R_1)^2}{8Fo_1(1-\varphi_0)}\right] \times W_{n-\frac{m}{2}-\frac{1}{4},\frac{m}{2}+\frac{1}{4}}\left(\frac{(r/R_1)^2}{4Fo_1(1-\varphi_0)}\right)\right].$$

Figure 3 shows the solutions (17) and (18) for two cases, when $Fo_1 \leqq Fo_0^*$ ($\tau \leqq \tau_0$) and $Fo_1 > Fo_0^*$ ($\tau > \tau_0$).

## NOTATION

$\Theta_i(r, 0, \tau) = T_i(r, 0, \tau) - T_0$, excess temperatures on the surface of the semiinfinite body in the corresponding regions of r (see text); $R_2$, $R_1$, r, the outer and inner radii of the annular heater, and the distance from the center, respectively; $q_0$ constant heat flux desnity in the annular region $R_1 \leqq r \leqq R_2$ on the surface of the semiinfinite body during the time interval $0 \leqq \tau \leqq \tau_0$; $a$, $\lambda$, b, thermal diffusivity, thermal conductivity, and thermal activity of the body; z, $\tau$, distance normal to the surface of the body and time, respectively; $Ki = q_0R_2/\lambda T_0$, $Ki_1 = q_0R_1/\lambda T_0$, Kirpichev numbers; $Fo - a\tau/T_2^2$, $Fo_1 = a\tau/R_1^2$, $Ro_0 = a\tau_0/R_2^2$, $Fo_0^* = a\tau_0/R_1^2$, Fourier numbers; $U(\tau - \tau_0) = U(Fo - Fo_0) = U(Fo_1 - Fo_0^*)$, Heaviside unit step function; $K_R = R_1/R_2$, ratio of the inner and outer radii of the annular heater; $W_{k,\mu}(X)$, Whittaker function [4-8]; $(1/2)_m$, Pokhgammer symbol; $C_n^m = \binom{n}{m}$, binomial coefficients; $\Theta_i^* = \Theta_i(r, 0, Fo)/T_0$, dimensionless relative temperature; $\varphi_0 = \tau/\tau_0$, relative time; $\tau_0$, time duration of the constant intensity heat source.

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# THICKNESS OF THE LAYER OF SORPTION DEVELOPER IN CAPILLARY INSPECTION

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The article introduces and analyzes expressions for determining the thickness of the layer of developer applied to the inspected solid body for revealing surface flaws of different shape by methods of capillary flaw detection.

Various methods of capillary inspection (or capillary flaw detection) are classed in one group by the following traits. Firstly, they are all intended for revealing surface flaws. Secondly, their principle of functioning is based on the use of a luminescent or colored indicator liquid (penetrant) which, having been previously applied to the inspected surface and then removed from it, penetrates from flaws into the thin layer of powdery or suspension developer and forms on its surface a contrasting, visualized "trace" of the flaw. A recent theoretical investigation of the hydrodynamics of indicator liquids in processes of capillary flaw detection made it possible to construct a hydrodynamic theory of the stages of making flaws visible and to derive a number of formulas for evaluating the sensitivity threshold and the duration of inspection [1, 2]. It follows from the obtained results that in revealing blind surface flaws with the aid of a powdery sorption developer, there exists a maximal thickness of the layer of developer $h_{max}$, and when this is exceeded, flaws of a certain width of opening (or less) cannot be made visible any more. We will find the values of $h_{max}$ in revealing cracks with plane parallel and nonparallel walls, and also of cylindrical flaws.

Crack with plane parallel walls (Fig. 1a). If on an inspected solid surface there are flaws, then an indicator liquid applied to it penetrates into their cavities under the effect of capillary pressure $p_c = 2\sigma \cos\theta/H$. As a rule, the penetrants wet solid surfaces well, and therefore for the sake of simplicity of further explanation we put $\cos\theta \approx 1$. After the liquid has been removed from the surface, the residual depth to which it fills the crack with depth $l_0$ and width H is determined by the expression $l = n l_0 \psi$ $(0 < n \leq 1)$ [1], where $\psi = 2\sigma/(2\sigma + p_a H)$. Assume that to the inspected surface a layer of powdery sorption developer with thickness h is applied (Fig. 1a). The penetrant from the cavity of the flaw wets the developer, and as a result a "trace" of the flaw forms on the outer surface of the developer which becomes luminescent in ultraviolet light. In dependence on the ratio between the

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